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Predicting Nonlinear Effects in Superconducting Microwave Transmission Lines from Mutual Inductance Measurements

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Abstract—We demonstrate the use of a new experimental technique based on mutual inductance measurements to quantitatively predict nonlinear effects in microwave devices fabricated from high temperature superconductor (HTS) materials. The mutual inductance measurements yield the current dependence of the penetration depth $\lambda(J)$ in unpatterned HTS thin films. This information is used to calculate third-harmonic generation in coplanar waveguide (CPW) transmission lines and compares very well with actual measurements of CPW transmission lines of variable dimensions fabricated from $YBa_2Cu_3O_{7-\delta}$ thin film samples. The mutual inductance measurements should prove extremely valuable as a screening technique for microwave applications of HTS materials that require very low nonlinear response.

I. Introduction

Microwave circuits fabricated from high temperature superconductor (HTS) materials often suffer from detrimental nonlinear effects, such as intermodulation distortion [1],[2] and harmonic generation [3]. The practical result of this nonlinear response is the creation of unpredictable interference signals within the frequency band of interest for many microwave applications. These nonlinear effects can seriously compromise the utility of microwave devices fabricated from HTS materials. What is urgently needed is an experimental technique that can be used to predict the nonlinear response of HTS microwave devices based on the starting material, prior to device fabrication.

In order to study the nonlinear response of HTS devices, we have developed a model nonlinear system, consisting of coplanar waveguide (CPW) transmission lines fabricated from $YBa_2Cu_3O_{7-\delta}$ (YBCO) thin films grown by pulsed-laser deposition. The nonlinear response of transmission lines of different geometries is determined from measurements of the generated third-harmonic signal as a function of incident power. This model nonlinear system allows us to perform detailed characterization measurements after every step in the device fabrication process, from film growth through device patterning and nonlinear evaluation. We have used this model nonlinear system to determine the relative importance of different processing variables, such as film deposition conditions [4], patterning methods, and device geometry [5], in minimizing nonlinear response. As a result of our investigation, we have identified a low frequency mutual inductance measurement for use as a screening technique for evaluating the nonlinear response of HTS materials prior to device fabrication [6]. We demonstrate how these mutual inductance measurements can be used to predict the nonlinear response of patterned devices of variable geometries at microwave frequencies.

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II. Mutual inductance measurements

We grow superconducting YBCO thin films by pulsed-laser deposition on $LaAlO_3$ substrates up to 15 mm x 15 mm. Before patterning the thin film samples into coplanar waveguide devices, we perform a number of characterization measurements to help determine the film quality. Sapphire dielectric resonator measurements yield the surface resistance, which at 76 K is typically in the range 250 to 300 $\mu\Omega$ when scaled to 10 GHz. We also use mutual inductance measurements to determine the transition temperature (typically 90-91 K) and the critical current density (typically 3-3.5 x 10^6 A/cm^2 at 76 K).

In addition to these characterization measurements, we exploit a new technique based on mutual inductance measurements to determine the dc current dependence of the penetration depth $\lambda(J)$ [6]. Such measurements are motivated by recent theoretical work that shows that a current-dependent penetration depth can lead to nonlinear effects in microwave devices [7]. These calculations assume a quadratic dependence of $\lambda(J)$ in order to calculate the third-order nonlinear products, but until now there has not been any direct experimental confirmation of this form for $\lambda(J)$. Figure 1 shows the results of the mutual inductance measurements of $\lambda(J)$ for a 50 nm YBCO thin film at 76 K. These measurements show that the dependence of λ on dc current density is indeed quadratic, at least up to current densities approaching the critical current density J_c (which is determined from mutual inductance measurements to be approximately

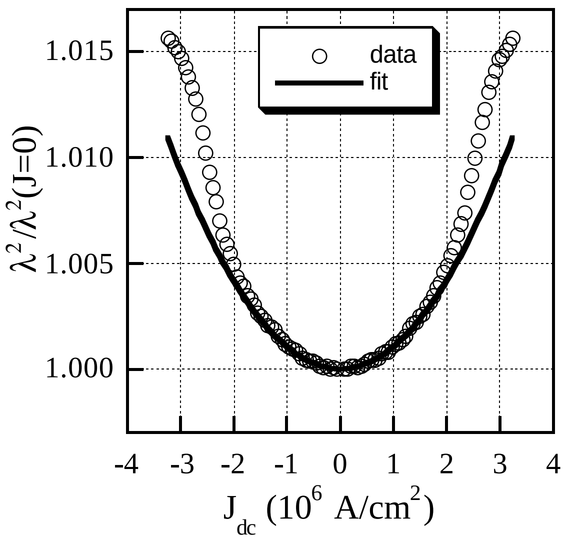

Fig.1. Current dependence of the penetration depth for a 50 nm YBCO film at 77 K, obtained using a 10 kHz mutual inductance technique.

3.4 x 10^6A/cm^2 for this sample). We fit the data in the low-current region of Fig. 1 to the form

$$\lambda^2(T,J) = \lambda^2(T)\left[1+\left(\frac{J}{J_0(T)}\right)^2\right] \qquad (1)$$

and obtain J_0 = 3.1 x 10^7 A/cm^2 at 77 K. Similar measurements on a thicker film (400 nm) grown by pulsed-laser deposition under slightly different growth conditions yield a similar value of J_0 = 3.5 x 10^7 A/cm^2 at 77 K. These results are significant because, if this λ(J) response is the dominant source of nonlinear effects, the theoretical analysis of Dahm et al. [7],[8] can be used to calculate the third-order nonlinear products of a wide range of different devices fabricated from these materials. These measurements also yield the value of the penetration depth for zero applied current density λ(J=0), which is important for calculating both the linear and nonlinear response of microwave devices.

III. TRANSMISSION LINE MODEL

We use these results for λ(J) to calculated the nonlinear (current-dependent) inductance per unit length $L(I) = L_0 + L'\cdot I^2$ present in planar structures, following Dahm [7]. We have generalized the analysis of Dahm to calculate the third-harmonic signal generated by a short length of HTS transmission line at microwave frequencies, instead of calculating the intermodulation products in a resonant structure. The details of the calculation of the third-harmonic signal as a function of incident power (P_{inc}) are presented elsewhere [5]; we quote here just the result for the third-harmonic signal P_3:

$$10\log_{10}(P_3) = -2\cdot IP_3 + 3*10\log_{10}(P_{inc}) \ . \qquad (2)$$

If we plot the measured power in the third-harmonic as a function of incident power on a log-log scale, (2) predicts that the P_3 data will have a slope of 3 with an intercept point $-2IP_3$. The quantity IP_3 is called the third-order intercept. It corresponds to the point where a line of slope 3 fit through P_3 would intercept a line of slope 1 fit through the fundamental; see Fig. 2 for an example [9]. The third-order intercept at frequency ω for a transmission line of length ℓ, center linewidth w, thickness t, and characteristic impedance Z_0 is given by the following expression (in dBm) [5]:

$$IP_3 = 10\log_{10}\left(\frac{2J_0^2(T)}{\mu_0\omega\lambda^2(T)}\frac{w^2t^2Z_0^2}{\Gamma\ell}\right) + 30 \qquad (3)$$

Γ is a geometrical factor that depends on the current distribution, and is given by [5]

$$\Gamma = \frac{w^2t^2\int J^4\,dS}{\left(\int J\,dS\right)^4} \ . \qquad (4)$$

The third-harmonic response calculated from (2) and (3) is strictly valid for transmission lines that have a length ℓ that is smaller than the effective wavelength so that the lumped-element approach used in [5],[7] is valid. We have calculated the effect of connecting together a series of such small elements to create a transmission line of arbitrary length L. As long as the third-harmonic signal remains much smaller than the fundamental ($P_3 \ll P_1$), the third-harmonic signal for the individual elements all add in phase, and the result (3) is valid for any transmission line length [10].

Figure 2 shows an example of the measured third-harmonic signal at 76 K as a function of incident power for a CPW transmission line fabricated from a 400 nm YBCO thin film. The CPW transmission line has a center conductor linewidth of 105 µm and a length of 6.54 mm, and has a nominal characteristic impedance of 50 Ω. The solid lines in Fig. 2 are fits of slope 1 and 3 to the fundamental and third-harmonic data, respectively. We compare the values for IP_3 extracted experimentally using (2) with the values for IP_3 calculated from (3) using inductively determined values for J_0 and λ, along with the transmission line's dimensions. For this CPW device, using J_0 = 3.5 x 10^7 A/cm^2 and λ = 312 nm (Γ is calculated to be 3.03 x 10^{11} m^{-2}), we calculate IP_3 = 82.7 dBm. This value compares extremely well with the experimentally determined value of IP_3 = 82.7 dBm, particularly considering that (3) uses only experimentally determined quantities (no adjustable parameters) to calculate IP_3.

The inset of Fig. 2 shows the saturation of P_3 observed as the incident power becomes high. At the point where P_3 begins to deviate noticeably from the "slope 3" behavior

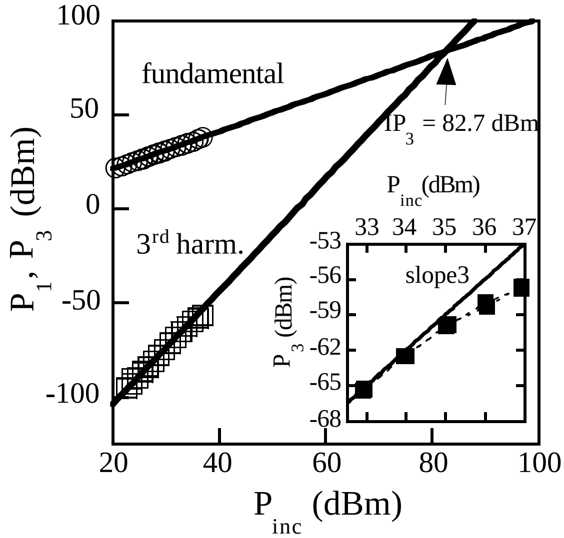

Fig. 2. Measured third-harmonic signal vs. incident power for a 105 μm wide, 6.54 mm long CPW transmission line. The device was fabricated from a 400 nm YBCO thin film. The solid line through the fundamental (P_1) data is a fit to a line of slope 1 and the solid line through the third-harmonic (P_3) data is a fit to a line of slope 3. The inset shows the deviation from slope 3 that occurs for high incident powers.

($P_{inc} \approx 35$ dBm), we estimate the peak rf current density to be 3.65×10^6 A/cm^2 (the average rf current density for this power is 8.5×10^5 A/cm^2). This deviation from "slope 3" behavior occurs because the shape of $\lambda(J)$ is no longer quadratic for such large peak current densities (see Fig. 1). In this case, more terms are needed in the expansion of $\lambda(J)$ in (1), and the third-harmonic vs. incident power data will in general no longer obey the "slope 3" behavior, which results from a purely quadratic form for $\lambda(J)$.

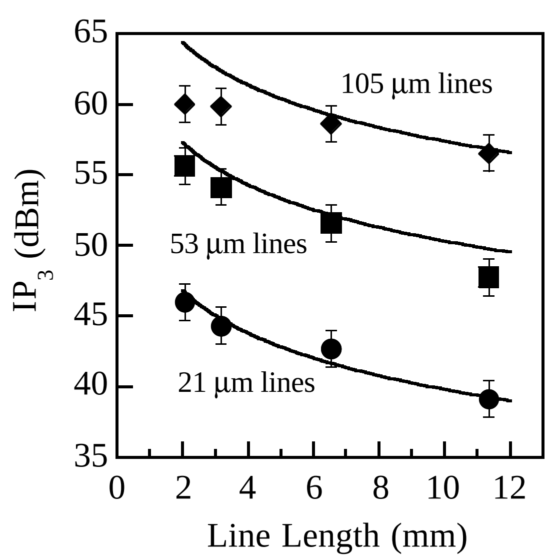

Fig. 3. Third-order intercepts vs. line length for an HTS transmission line with a 5 GHz fundamental signal. The center conductor linewidths are 21 μm, 53 μm, and 105 μm. The solid lines are calculated from (3), with $\lambda(77\text{ K}) = 354$ nm and $J_0(77\text{ K}) = 3.1 \times 10^7$ A/cm^2. The error bars represent the standard deviation of IP_3 determination across the chip.

IV. Third-Harmonic Response for Different Geometries

To confirm that the third-harmonic response predicted by (2) and (3) is generally valid, we determine the third-order intercept point IP_3 at 76 K for a number of different transmission line geometries, as shown in Fig. 3. These data are for 11 different CPW transmission lines fabricated from a single 50 nm YBCO thin film, with different linewidths and lengths. As this thin film sample was grown under nominally identical deposition conditions to the sample of Fig. 1, we expect the inductively measured values of $J_0(77\text{ K}) = 3.1 \times 10^7$ A/cm^2 and $\lambda(77\text{ K}) = 354$ nm to accurately describe the resulting third-harmonic generation of all devices on this chip. This is precisely what is observed in Fig. 3, which shows as solid lines the prediction of (3), *based solely on the inductively measured values of λ and J_0 and the transmission line dimensions.* These data illustrate the power of the inductive results for predicting the nonlinear response of microwave devices of arbitrary geometry.

V. Conclusions

We have developed a new screening technique for predicting the nonlinear response of microwave devices fabricated from HTS materials. The technique uses low-frequency (10 kHz) mutual inductance measurements of unpatterned superconducting films to determine the current dependence of the penetration depth $\lambda(J)$ [6]. These mutual inductance measurements predict third-harmonic generation in CPW devices of different geometries patterned from YBCO thin films. This technique provides a valuable method for determining the nonlinear response of HTS microwave devices prior to circuit fabrication.

V. References

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